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Suboptimal Command Generation for Control Moment Gyroscopes and Feedback Control of Spacecraft

S. R. Vadali* and S. Krishnan†
Texas A&M University, College Station, Texas 77843-3141

This paper presents a suboptimal gimbal-angle command generation technique and a Lyapunov-based tracking control law for control moment gyroscopes. The formulation of the open-loop maneuver problem exploits the principle of conservation of angular momentum. This allows for a reduction in the dimensionality of the problem—only the kinematics need be considered. The gimbal rates are parametrized as polynomial functions of time, and the parameters are optimized. The objective function of the optimization is the singularity measure. The optimal null motion used is determined after computing the open-loop gimbal-rate profiles. A feedback strategy is used to track the open-loop reference attitude and angular velocity profiles. Null motion is also used to allow the closed-loop gimbal angles to follow the open-loop gimbal-angle histories. This procedure avoids neighboring singularities and also provides repeatability. Simulation results are presented for rest-to-rest maneuvers using a model of a ground-based test article. The results show that a wide domain of singularity-free operation around the reference solution can be provided with this approach.

Introduction

SINGLE-GIMBAL control moment gyroscopes (CMGs) have a number of advantages as spacecraft attitude control devices.[1] Their main disadvantage is that for any system of n CMGs, there exist 2^n sets of singular gimbal angles for which no output torque can be generated along particular axes. This is also true for double-gimbal CMGs, but the problem is less severe. Both open-loop and feedback command generation techniques for CMGs have been studied extensively in the literature. Margulies and Aubrun[1] presented a geometrical approach to classifying various types of singularities. Cornick[2] suggested null-motion approaches to avoiding singular gimbal configurations. Null motion is a motion of the gimbals that produces no net torque on the spacecraft. The amount of null motion to be added is a function of the entire maneuver (global) history, and adding the right amount at the right time is the key to successful singularity avoidance.

There are a number of similarities between CMGs and redundant-link manipulators. In the robotics literature, Nakamura and Hanafusa[3] developed the singularity-robust inverse approach for obtaining an approximate solution for the joint angular velocities in the vicinity of singularities. Bedrossian[4] used this approach on the CMG problem. Vadali et al.[5] showed the existence of, and a method of determining, preferred initial gimbal angles that avoid singularities for a class of maneuvers with known torque requirements. Feedback control and steering laws for maneuvering spacecraft using CMGs are presented by Oh and Vadali.[6] The above methods are classified as local methods. However, a general steering law with guaranteed singularity avoidance properties has not been demonstrated yet. In fact, it has been shown by Wampler et al.[7,8] that no local method can successfully avoid singularities and a global method must be used for this purpose. Paradiso[9] presented a directed-search approach for global steering of the gimbal angles. Hoelscher and Vadali[10] present an open-loop optimization technique using the conjugate-gradient method. The above techniques can successfully avoid singularities but are demanding in terms of processing complexity.

In the robotics literature, the concept of drift-free motion or repeatable joint-space trajectories has been developed. Repeatability allows the joint angles to follow closed paths as the end effector moves between two positions. This concept also has a parallel for the CMG problem in that, in the absence of disturbances, it is desirable for the gimbal angles to return to their preferred initial conditions at the end of a rest-to-rest maneuver. Shamir and Yomdin[15] developed a test for determining whether a given trajectory in the joint space is repeatable or not. The test involves the calculation of a Lie-bracket condition. Generally the pseudoinverse steering law does not satisfy this condition. Roberts and Maciejewski[16,17] used the Lie-bracket condition to propose a repeatable-control law. Unfortunately, this technique requires extensive computations, which are impractical for real-time implementation.

This paper presents a suboptimal gimbal-angle command generation technique for CMGs. The gimbal rates are parametrized as polynomial functions of time. The coefficients of the polynomials become parameters to be optimized. The objective function of the optimization is the singularity measure. Momentum and attitude constraints can also be imposed if necessary. Once the gimbal rates are obtained, the torque commands are easily computed. It is also possible to compute the amount of null motion added by the optimization process. A feedback strategy is used to track the reference attitude and angular velocity profiles in a Lyapunov sense. In the absence of external torques, null motion is also used to position the gimbals at their preferred starting values to provide repeatability. This control law is quite simple to implement in real time. Simulation results are presented using a model of a ground-based test article.

Formulation

The formulation of the spacecraft maneuver problem exploits the principle of conservation of angular momentum. In the absence of external torques, the angular momentum of the spacecraft is conserved. This constraint allows for a reduction in the dimensionality of the problem—only the kinematics of the spacecraft and CMGs need be considered.

The equations of motion can be compactly written in the following form:

$$\dot{\boldsymbol{\theta}} = S(\boldsymbol{\theta})\boldsymbol{\omega} \tag{1}$$

$$R^T(I\boldsymbol{\omega} + \boldsymbol{h}) = \boldsymbol{H} = \text{const} \tag{2}$$

where $\boldsymbol{\theta}$ is the vector of Euler angles representing the spacecraft attitude, $\boldsymbol{\omega}$ is the angular velocity vector, R is the direction-cosine matrix orienting the body axes with respect to the inertial axes, I is the inertia matrix of the spacecraft including the CMGs, $\boldsymbol{h}$ is the angular momentum of the CMGs, and $\boldsymbol{H}$ is the total inertial angular momentum. As shown later, Euler parameters can also be used instead of Euler angles to represent the spacecraft attitude. Equation (2) can

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*Professor, Department of Aerospace Engineering. Associate Fellow AIAA.

†Graduate Student, Department of Aerospace Engineering. Student Member AIAA.

be used to express the angular velocity of the spacecraft in terms of the angular momentum of the CMGs and the spacecraft attitude. This concept has been used for reaction-wheel control systems by Vadali and Junkins.[11] Applications of this concept to robotic manipulators can be found in Reyhanoglu and McClamroch,[12] Mukherjee and Zurowski,[13] and Krishnan and Vadali.[14] The CMG angular momentum vector takes the following form for a system of four CMGs arranged in the pyramid configuration[4,5]:

$$h(\sigma) = h \begin{bmatrix} -\cos\delta\sin\sigma_1 - \cos\sigma_2 + \cos\delta\sin\sigma_3 + \cos\sigma_4 \\ \cos\sigma_1 - \cos\delta\sin\sigma_2 - \cos\sigma_3 + \cos\delta\sin\sigma_4 \\ \sin\delta\sin\sigma_1 + \sin\delta\sin\sigma_2 + \sin\delta\sin\sigma_3 + \sin\delta\sin\sigma_4 \end{bmatrix} \tag{3}$$

The gimbal angles are denoted by σ_i. Each CMG is assumed to have the same angular momentum h, and the pyramid angle is δ. The torque $\boldsymbol{\tau}$ produced by the CMGs is related to the gimbal rates as follows:

$$C\dot{\boldsymbol{\sigma}} = \boldsymbol{\tau} \tag{4}$$

The maneuver problem now becomes one of transferring the attitude from the initial state to the desired state using the following equation:

$$\dot{\theta} = S(\theta)I^{-1}[RH - h(\sigma)] \tag{5}$$

The boundary conditions on ω transform to boundary conditions on $\boldsymbol{\sigma}$, but these are not unique. However, one can select a preferred set of boundary conditions to achieve other objectives such as singularity avoidance.[5] The gimbal rates are used as control variables, since they are proportional to the torque commands. A parameter optimization approach is utilized to obtain suboptimal gimbal rate commands. For example, if the torque is required to be zero at the beginning and the end of a rest-to-rest maneuver, the gimbal rates can be parametrized as follows:

$$\dot{\sigma}_i = p_{1i}t + p_{2i}t^2 + p_{3i}t^3 + p_{4i}t^4, \qquad i = 1, \dots, 4 \tag{6}$$

$$p_{1i} + p_{2i} + p_{3i} + p_{4i} = 0 \tag{7}$$

In the above formulation, the final time is conveniently nondimensionalized to 1. This parametrization forces the gimbal rates to be zero at the initial and final times. The initial gimbal angles are specified by the boundary conditions as discussed before. If additional smoothness is desired, higher-order polynomial functions can be used to represent the gimbal rates.

The parameters p_{ij} in Eq. (6) are determined to minimize or maximize:

$$J(\sigma) = \max_P \min_t \det(CC^T) \tag{8}$$

where $C = \mathrm{d}\boldsymbol{h}/\mathrm{d}\boldsymbol{\sigma}$ is the Jacobian matrix and P is the set of all the parameters to be optimized. In other words, the objective is to maximize, with respect to the parameters, the minimum value, over the entire maneuver duration, of the singularity measure. In the above equation, the determinant is an indicator of how close the CMG cluster is to a singular point. It should be emphasized that by virtue of the above performance index, internal singularities are avoided. The external singularity corresponds to momentum saturation, which is avoidable by suitable momentum management. In certain cases, however, the solution for the gimbal rates might violate hardware constraints such as the maximum gimbal rate. In such cases, a gimbal-rate inequality constraint can be imposed and the final time can be incorporated as an extra free parameter to be selected. This class of optimization problems can be conveniently solved using the sequential quadratic programming[18] (SQP) approach.

Determination of Null Motion

The null motion used by a control law or an open-loop control can be easily determined by comparing the gimbal rates obtained to the respective gimbal rates produced by the pseudoinverse control law, which does not use any null motion. Starting with the CMG torque equation (4), the pseudoinverse steering law computes the gimbal rates $\dot{\boldsymbol{\sigma}}_p$ as follows:

$$\dot{\boldsymbol{\sigma}}_p = C^T(CC^T)^{-1}\boldsymbol{\tau} = C^\dagger\boldsymbol{\tau} \tag{9}$$

The general solution to Eq. (4) is

$$\dot{\boldsymbol{\sigma}} = C^\dagger\boldsymbol{\tau} + \hat{\boldsymbol{n}}a \tag{10}$$

where $\hat{\boldsymbol{n}}$ is the unit null vector spanning the null space of C. The amount of null motion is denoted by the scalar a. The unit null vector can be determined analytically.[4] One can also obtain the amount of null motion using Eqs. (4), (9), and (10) as follows:

$$\hat{\boldsymbol{n}}a = [I - C^T(CC^T)^{-1}C]\dot{\boldsymbol{\sigma}} \tag{11}$$

where I is the identity matrix. Since a is a scalar, it can be obtained by dividing any component of the right-hand side of Eq. (11) by the corresponding (nonzero) component of the unit null vector.

Feedback Control

Computation of the open-loop command discussed above is based on the kinematics of the spacecraft and the CMGs. It is also noted that the initial conditions used in the optimization may not always be the same as that present in a real application. Hence there is a definite need for feedback control. To exploit the availability of the precomputed open-loop command that definitely avoids singular regions, a tracking control law is developed in this section. As long as the deviations of the actual trajectories from the reference trajectory are not too large, the singularity problem will not arise. The development of the control law is presented in terms of Euler parameters instead of Euler angles for compactness. The differential equations for the time rate of change of these parameters is given by

$$\dot{\beta} = \tfrac{1}{2}H^T(\beta)\omega \tag{12}$$

where

$$H(\beta) = \begin{bmatrix} -\beta_1 & \beta_0 & \beta_3 & -\beta_2 \\ -\beta_2 & -\beta_3 & \beta_0 & \beta_1 \\ -\beta_3 & \beta_2 & -\beta_1 & \beta_0 \end{bmatrix}$$

It is useful to write the above equation in its inverted form for estimating the angular velocities from quaternion measurements:

$$\omega = 2H(\beta)\dot{\beta} \tag{13}$$

The equation of rotational motion of a spacecraft is given by the following:

$$I\dot{\omega} = -\tilde{\omega}(I\omega + \boldsymbol{h}) - C\dot{\boldsymbol{\sigma}} \tag{14}$$

where $\tilde{\omega}$ is the angular velocity cross-product matrix. The application of Lyapunov stability theory to maneuver rigid and flexible spacecraft has been demonstrated in many works. The unified framework for obtaining final position and tracking control laws using this procedure are summarized below.[19]

Let β_{ref} and ω_{ref}, respectively, be the reference quaternion and the reference angular velocity profiles to be tracked. By construction, the reference angular velocity and CMG momentum satisfy Eq. (14). An error vector is created with respect to the current states as follows:

$$\begin{bmatrix} \boldsymbol{e}_\beta \\ \boldsymbol{e}_\omega \end{bmatrix} = \begin{bmatrix} \beta - \beta_{\mathrm{ref}} \\ \omega - \omega_{\mathrm{ref}} \end{bmatrix} \tag{15}$$

To drive this error to zero, a Lyapunov function V is defined as

$$V = k\boldsymbol{e}_\beta^T\boldsymbol{e}_\beta + \tfrac{1}{2}\boldsymbol{e}_\omega^T\boldsymbol{e}_\omega \tag{16}$$

where k is a positive scalar. The derivative of this function is given as follows:

$$\dot{V} = \boldsymbol{e}_\omega^T[-H(\beta)\beta_{\mathrm{ref}} - I^{-1}[\tilde{\omega}(I\omega + \boldsymbol{h}) + C\dot{\boldsymbol{\sigma}}] - \dot{\omega}_{\mathrm{ref}}] \tag{17}$$

The properties $H(\beta)\beta_{\text{ref}} = -H(\beta_{\text{ref}})\beta$ and $H(\beta)\beta = \mathbf{0}$ have been used to arrive at the above result. The feedback control law can be derived as given below:

$$C\dot{\sigma} = -kIH(\beta)\beta_{\text{ref}} + k_1 I e_\omega - \tilde{\omega}(I\omega + h) - I\dot{\omega}_{\text{ref}} \equiv \tau \quad (18)$$

where k_1 is a positive scalar and τ is the control torque by definition. It is more practical to rewrite the above equation by replacing $I\dot{\omega}_{\text{ref}}$ with the reference quantities in the right-hand side of Eq. (14). It can be easily shown by substituting Eq. (18) into Eq. (17) that the above control law renders $\dot{V}$ negative semidefinite. Further analysis using the invariance theorem assures the global asymptotic stability of the closed-loop system[19] if singularities are not encountered. This is assured if the deviation of the initial attitude is not too large.

Steering Laws for Repeatability

Use of the pseudoinverse steering law results in arbitrary final gimbal angles unlike those obtained by using the open-loop control. It is therefore conceivable that if a slewing maneuver is repeated several times, the gimbal angles will drift away from their preferred settings and possibly attain a singular configuration. Hence it is advantageous to implement a closed-loop steering law that not only provides the required torque, but also forces the gimbals to a desired orientation at the end of the maneuver. This will reduce the likelihood of reaching singularities even when the CMGs are used for extended periods of time. In order to achieve this objective, the pseudoinverse steering law is modified to include null motion as described by Eq. (10). The scalar a is made proportional to the error between the open-loop reference and the instantaneous gimbal angles as follows:

$$a = -k_2\hat{n}^T(\sigma - \sigma_R) \quad (19)$$

where k_2 is positive scalar and σ_R indicates the open-loop reference gimbal angle vector. There is a possibility of gimbal lock when the null vector and the gimbal-angle error vector in Eq. (19) are orthogonal to each other. This is especially important if the reference-gimbal-angle history is inconsistent with respect to the angular momentum of the system. In such situations, which might arise from external disturbances or nonzero initial angular velocities, gimbal lock prevents the gimbals from moving around after the maneuver has been completed. However, for the examples considered during this study, it has been determined numerically that there exists a large region around any desired nonsingular gimbal angle state where Eq. (19) can be used successfully to achieve repeatability.

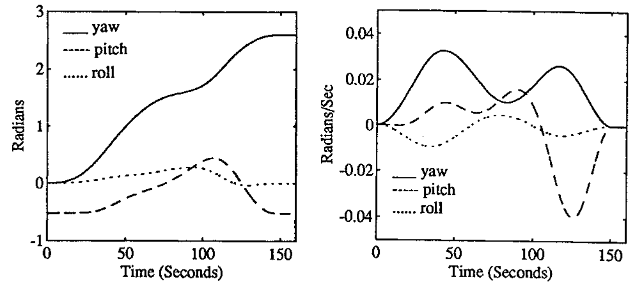
Model of the Spacecraft

The simulation results presented in this paper are based on a rigid-body model of the Advanced Space Structures Research Experiments (ASTREX) test article. This test article is designed to float on a 5-m pedestal using a two-axis air-bearing system. Its mass is approximately 4000 kg (Ref. 20). The primary structure is a 5.5-m truss that supports the thrusters and the four CMGs. The momentum of each CMG is 542.32 N · m · s, and the pyramid angle δ is 15 deg. The maximum gimbal rate is 2 rad/s. The test article nominally rests in a 30-deg nose-down attitude. The approximate moment of inertia matrix with respect to the basic coordinate system located at the pivot point (units: kg · m^2) is given below:

$$I = \begin{bmatrix} 18{,}941 & -25 & -243 \\ -25 & 11{,}804 & 25 \\ -243 & 25 & 14{,}188 \end{bmatrix}$$

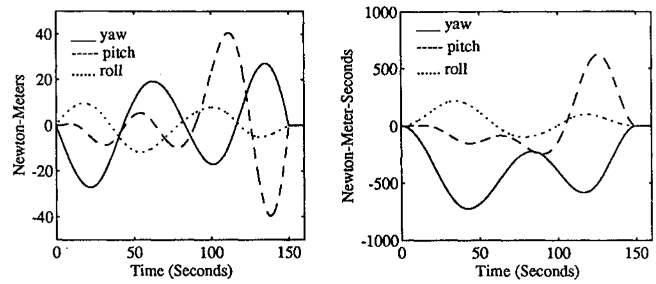
Open-Loop Optimization Results

A yaw maneuver simulation is considered. The initial Euler angles are [0, −30, 0] deg, and the final Euler angles are [150, −30, 0] deg. The maneuver time is selected to be 150 s. The initial gimbal angles are assumed to be at [0, 0, 0, 0]. Figures 1a and 1b show, respectively, the attitude and angular velocity profiles. The CMG torque and momentum profiles are shown in Figs. 2a and 2b. Figures 3a and 3b show, respectively, the gimbal angles and gimbal rates. Figure 4 shows the singularity measure [det(CC^T)]. Figures 5a and 5b show



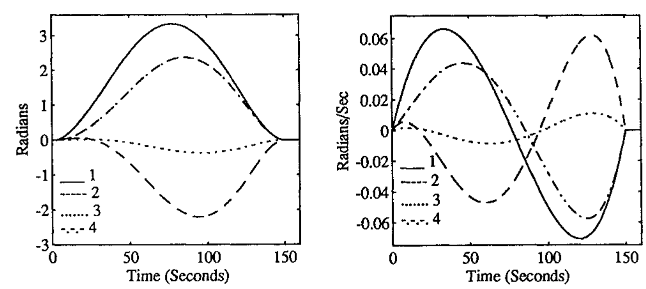
a) Attitude profile b) Angular velocity profile

Fig. 1 Open-loop attitude and angular velocity profiles.



a) Torque profile b) Momentum profile

Fig. 2 Open-loop torque and momentum profiles.



a) Gimbal angle profile b) Gimbal rate profile

Fig. 3 Open-loop gimbal angle and gimbal rate profiles.

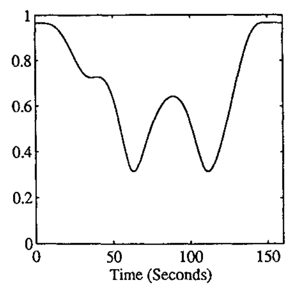
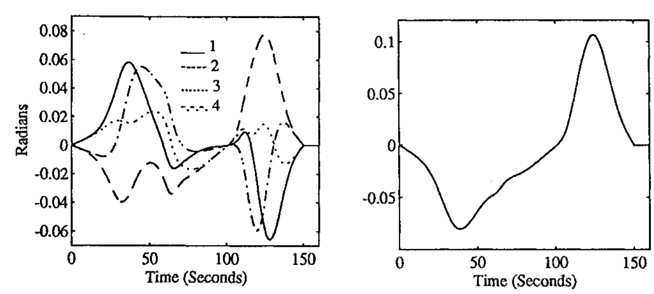


Fig. 4 Open-loop singularity measure.



a) Null motion rates b) Amount of null motion

Fig. 5 Open-loop null motion.

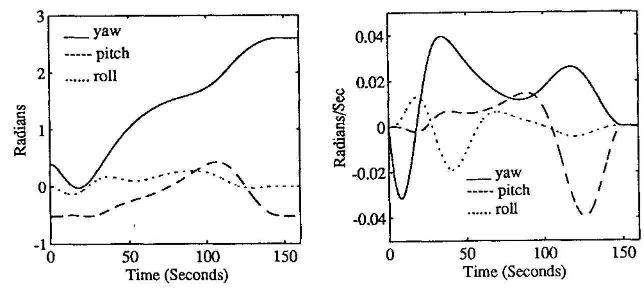
the null gimbal rates and the amount of null motion added. The gimbal angles return to zero as desired. It is seen from Fig. 4 that there is a sufficient singularity margin during the maneuver. The optimal maneuver uses a significant amount of null motion. The null gimbal rates are of the same levels as the total gimbal rates (Fig. 3b). It is also interesting that the direction of the null motion changes sign roughly at the midpoint of the maneuver.

Closed-Loop Control

Simulation results using the pseudoinverse control law (9) for tracking the reference trajectories of the previous example are given in Figs. 6–9. The values of the gains in the control and steering laws are the following: $k = 0.02$ N/kg · m, $k_1 = 0.1$ N · s/kg · m, and $k_2 = 0.5\ \mathrm{s}^{-1}$.

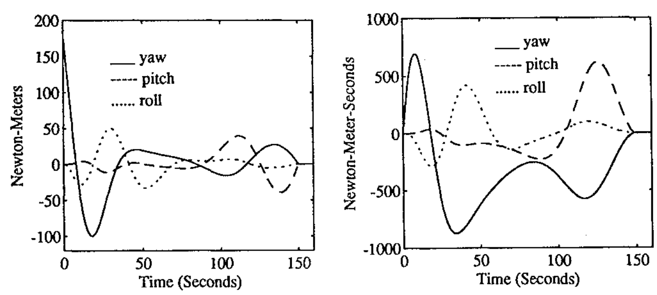
A rest-to-rest maneuver is considered with an initial attitude deviation of 20 deg in yaw from that used in the open-loop optimization. The maximum initial yaw deviation that can be allowed with the current open-loop profile is approximately 22 deg, based on simulations with different values of the initial error and observing the minimum value of the singularity measure. It is also noted that the singularity encountered is not internal but saturation. The initial torque level is high because of the yaw error, but the gimbal rates are well within their limits. Note that there is a large singularity margin during the maneuver; hence singularities are successfully avoided. Figure 8a shows that the final gimbal angles are not the same as their respective starting values and the singularity measure at the end of the maneuver is lower than its starting value. It is clear that if the same yaw maneuver were performed again and again, the final gimbal angles might reach undesirable (singular) states.

Simulation results for the null-motion steering law [Eqs. (10) and (19)] are given in Figs. 10–12. Note that after the transients have subsided, the closed-loop gimbal angles in Fig. 10 closely resemble the open-loop gimbal-angle profiles in Fig. 3.



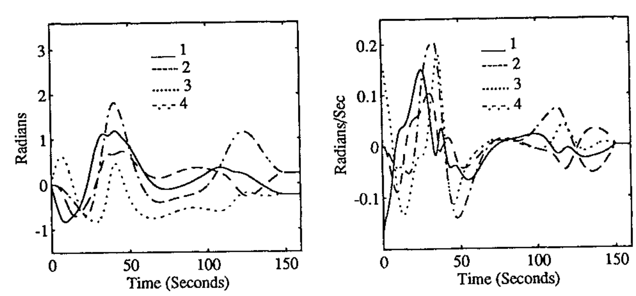
a) Attitude profile b) Angular velocity profile

Fig. 6 Closed-loop attitude and angular velocity profiles.



a) Torque profile b) Momentum profile

Fig. 7 Closed-loop torque and momentum profiles.



a) Gimbal angle profile b) Gimbal rate profile

Fig. 8 Closed-loop gimbal angle and gimbal rate profiles (no null motion).

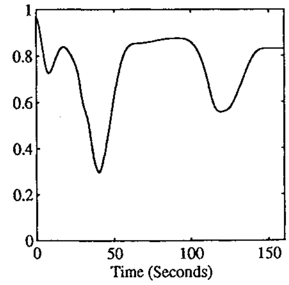
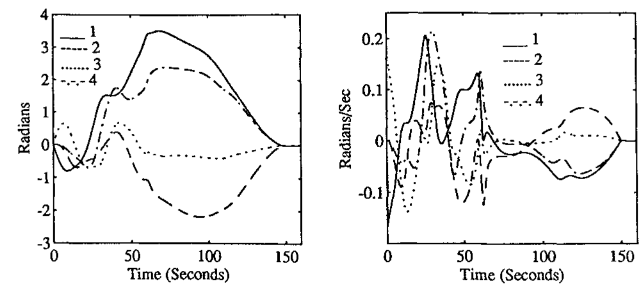


Fig. 9 Closed-loop singularity measure (no null motion).



a) Gimbal angle profile b) Gimbal rate profie

Fig. 10 Closed-loop gimbal angle and gimbal rate profiles (with null motion).

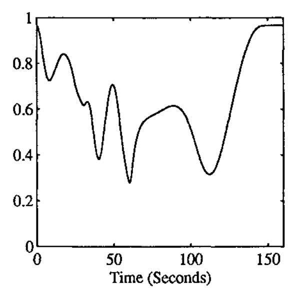
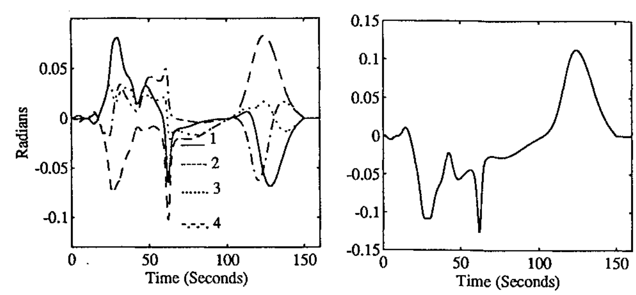


Fig. 11 Closed-loop singularity measure (with null motion).



a) Null motion rates b) Amount of null motion

Fig. 12 Closed-loop null motion.

Conclusions

This paper presents a new approach to open-loop command generation for single-gimbal control moment gyroscopes. The solutions are suboptimal, since the structure of the gimbal-angle profiles is parametrized a priori. These solutions can be obtained quite easily using a nonlinear programming technique. A method to determine the amount of null motion used by the open-loop control is presented. The open-loop trajectories are tracked using a Lyapunov stable tracking law with null motion. Null motion in the feedback application is used to obtain repeatability in the gimbal-angle space. Results of the open-loop and closed-loop simulations show that a wide domain of singularity-free operation around the reference solution can be provided with this approach.

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